Bonds and returns questions

Name

Institution

Date

BONDS AND RETURNS

QUESTION ONE

Convexity and bond duration measures the sensitivity of the price of bond compared to the changes in interest rates or yields. The bond price at present value is the discounted value of the expected future cash flows. Assuming the future cash flow remains constant, the price of bond will change as time elapses and with the changes in yield. The change in prices of bond, denoted as ΔP over time Δt is given by the following partial differential equation (PDE)

$$\Delta P = \frac{\delta P}{\delta t}\Delta t + \frac{\delta P}{\delta k}\Delta k + \frac{1}{2}\frac{\delta^2 P}{\delta k^2}(\Delta k)^2$$

The first derivative is the first term denoted as below

$$\Delta P = \frac{\delta P}{\delta t}\Delta t$$

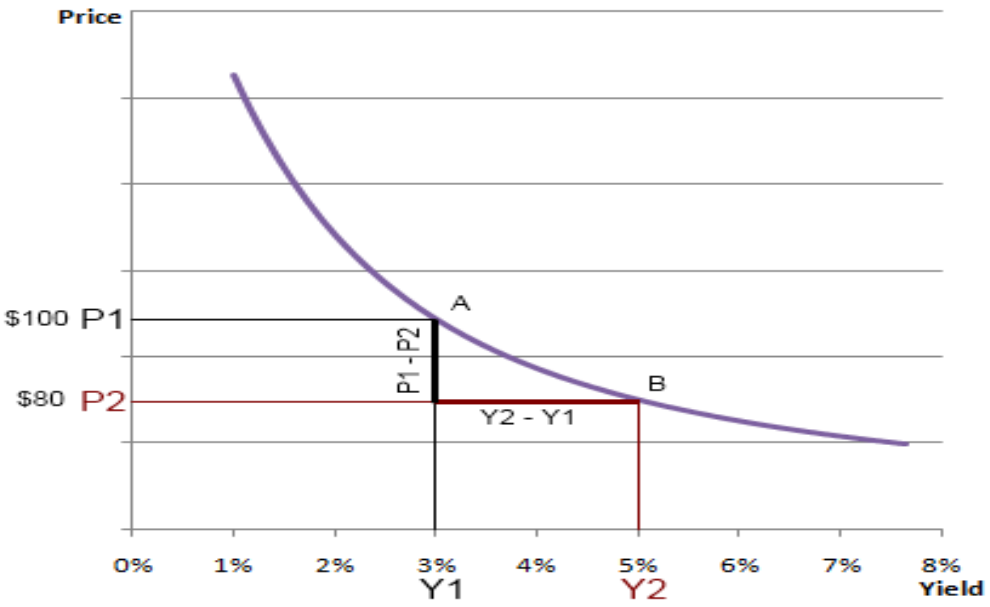
The change in price is measured by the partial derivative that occurs with change in time while the yield (k) is held constant. You can keep time constant and change the yield as shown below:

$$+ \frac{\delta P}{\delta k}\Delta k +$$

The second derivative related to the price of the bond is the partial differentiation equation shown below which is the last term of the PDE.

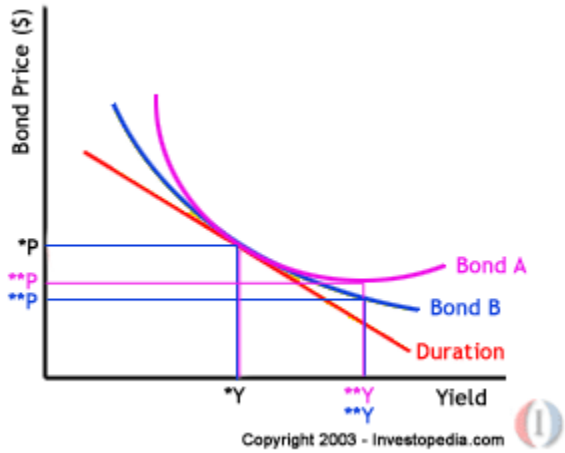
$$\frac{1}{2}\frac{\delta^2 P}{\delta k^2}(\Delta k)^2$$

Convexity is the second derivative of the price function. Convexity provides price predicting tool for bonds as it identifies the bonds interest rate risk and help investors consider whether the yield of the bond is worth the underlying risk. Convexity is the degree of the shape of the yield curve of a bond and the extent to which the price of the bond is sensitive to interest rate changes as shown below:



$$\textit{The first derivative (or the slope of the line joining A and B)} = \frac{P1 - P2}{Y1 - Y2}$$

In general, convexity assist potential bond buyers to anticipate the future price of bonds based on changes on interest rates. Generally, a fall in interest rate cause the price of bond to increase. However, for the cases of bonds with negative convexity, a fall in interest rate makes the bond lose its value. This often happens with Mortgage backed securities (MBS) since they rely on the underlying mortgage loans that are always paid early through refinancing when interest rates fall. The principal are paid early from prepayments leaving investors with the option of reinvesting their money at the current market rates that are lower.



The callable bonds also exhibit negative convexity at certain yields or prices. This is because the incentive of the issue to refer to the call of a bond at par will increase as interest rate falls. Consequently, the bonds price may decrease as it becomes clearer that the bond will be called.

The fixed income portfolio manager will use the concept of convexity to hedge against risk. Typically, increase interest rate makes the fixed rate income become less attractive to investors. A decrease in convexity of a bond will reduce the exposure to market interest rates and hence the fixed income will be considered hedged. In essence, higher yields or corporate rate implies the lower the market risk and convexity of a bond. The reduction in risk is caused by tremendous reduction in market rates to surpass the coupon on the bond and hence investor is exposed to less risk.

For risk neutral manager there's less risk to the investor when the bond has a high coupon or yield since market rates would have to increase significantly to surpass the bond's yield

QUESTION TWO

BONDS AND RETURNS

Bond duration refers to the measure of how price sensitivity of a bond or other types of debt instrument are changed by the interest rate. It's the amount of time it takes for bondholder to receive the principle value of bond. Higher duration is exposed to interest and credit risk. Lower level of duration is acceptable.

100 basis points is 1%

Coupon = 2.35%, Yield=2.76%, Time=10 years

Time	**Cash flow**	Discounting factor (2.76%)	PV	**(PV/Total)* Time**
1	$24	0.9731	$23.36	$0.0232549
2	$24	0.9546	$22.91	$0.0456240
3	$24	0.9327	$22.38	$0.0668647
4	$24	0.9113	$21.87	$0.0871059
5	$24	0.8904	$21.37	$0.1063824
6	$24	0.8699	$20.88	$0.1247278
7	$24	0.8499	$20.40	$0.1421747
8	$24	0.8304	$19.93	$0.1587546
9	$24	0.8114	$19.47	$0.1744982
10	$1,024	0.7927	$811.75	$8.0825682
Total			$1,004.32	$9.0119555

Based on the calculation on the table, the duration is 9.0119555 years

If the yield moves upwards by 100 basis points due to the shock, the following changes will be made

Face Value = $1000, Coupon= 2.35%, Yield= 3.76%, Time=10 years

Calculation of the duration will be given as below:

Time	**Cash flow**	Discounting factor (3.76%)	PV	**(PV/Total)* Time**
1	$24	0.9638	$23.13	$0.0230308
2	$24	0.9288	$22.29	$0.0443925
3	$24	0.8952	$21.48	$0.0641757
4	$24	0.8627	$20.71	$0.0824668
5	$24	0.8315	$19.96	$0.0993480
6	$24	0.8013	$19.23	$0.1148975

7	$24	0.7723	$18.54	$0.1291896
8	$24	0.7443	$17.86	$0.1422949
9	$24	0.7173	$17.22	$0.1542808
10	$1,024	0.6914	$707.95	$7.0490114
Total			$888.36	$7.9030881

After the shock of 100 basis points, the new duration is 7.9030881 years

If the basis point moves upwards 100 points, the change in percent yield witnessed is 0.6007%

If we experience a shock that moves the yield 100 basis points downwards

Face value = $1000, Yield=1.76%, Coupon=2.35%, Time=10 years

The duration is calculated as shown below:

Time	**Cash flow**	Discounting factor (1.76%)	PV	**(PV/Total)* Time**
1	$24	0.9827	$23.58	$0.0234835
2	$24	0.9657	$23.18	$0.0461546
3	$24	0.9490	$22.78	$0.0680345
4	$24	0.9326	$22.38	$0.0891437
5	$24	0.9165	$22.00	$0.1095024
6	$24	0.9006	$21.61	$0.1291302
7	$24	0.8850	$21.24	$0.1480463
8	$24	0.8697	$20.87	$0.1662694
9	$24	0.8547	$20.51	$0.1838179
10	$1,024	0.8399	$860.06	$8.5636096
Total			$1,058.22	$9.5271920

The duration for the yield is 9.52719 years if the shock causes the bond yield to move down by 100 basis points.

If the interest rates changes by 100 basis points downward, the percent change in yield becomes 0.5751%

In both the two cases, the bond is protected since the upward or downward movement of interest rate by 100 basis points has not reduced to the point below at 7.922 years.

QUESTION THREE

Assume future price is $1000, purchase price = $621.97, number of years = 14 years

Yield = $\{\frac{Future\ Value}{Purchase\ Price}\}^{1/n}$-1

$$\text{Yield} = \left(\frac{\text{Future Value}}{\text{Purchase Price}}\right)^{\frac{1}{n}} - 1$$

N= Years

Yield = $\{\frac{1000}{621.97}\}^{1/14}$-1 = 1.0345 -1 = 3.45%

Modified duration

Modified duration equals Macaulay's duration divided by (1 + y/m):

Modified Duration = t/(1+y/n) and

Where t is the time to maturity and y is the nominal yield while n is the number of compounding per annum

Modified Duration = 14 years/(1+3.45%) = 13.533 years

Modified duration = 13.533 years

Advantages and disadvantage of coupon bonds

Zero coupon bonds has many advantages that it can offer to their investors. The biggest benefit of a zero coupon bond is its ability to be predictable. If the bond is not sold before date of maturity, you will not worry of changes in market prices since the worth of the investment is known very well in advance. If the zero coupon is sold before date of maturity, you may lose money due to ups and downs in the markets. Zero coupon bonds can be customized to meet your needs. For example, if you know you will require money in 10 years when your child goes to college, you can purchase the bond for $621.97 for every $1000 that you will require assuming interest of 3.45%.

QUESTION FOUR

The fisher effect, a theory well developed by Irvin Fisher postulates that real interest rate are independent of monetary base changes. According to Fisher, real interest rate is nominal interest rate less inflation rate. The difference between nominal and real interest rate are affected

by inflation, a statement that is agreed by all economist but Fisher had a more forward looking interpretation. He postulated that an increase in inflation results to a fall in the interest rate unless the nominal interest rate increases at the same rate as the rate of inflation.

SUMMARY OUTPUT		
Regression Statistics		
Multiple R	0.497209859	
R Square	0.247217644	
Adjusted R Square	0.236905557	
Standard Error	0.005018215	
Observations	75	
ANOVA		
	df	*SS*
Regression	1	0.000603714
Residual	73	0.001838321
Total	74	0.002442035
	Coefficients	*Standard Error*
Intercept	0.039022092	0.002665829
X Variable 1	0.52343517	0.106904615

The Multiple R is a correlation coefficient that measures the strength of the linear relationship between treasury yield bill and inflation rate. From the extract of the regression equation, correlation coefficient, R is 0.4972 implying there is a less than average positive relationship between inflation rate and treasury yield bill with a lower standard error of 0.0026.

The R^2 which is the measure of goodness of fit indicates that only 24% of the values fit the regression analysis model. In other words, the 24% of the treasury T bills are explained by the inflation rate in total of 75 observations. In our data set, the treasury t bills is the y-axis and

the inflation rate is the x-axis and the linear regression formula based on the sample out will be as follows:

Y=0.523X-0.039

Therefore, there is a direct relationship between inflation rate and treasury bills and an increase in inflation rate will increase the treasury T-bills yield. The result contradicts Fisher effect which postulates that an increase in inflation results to a fall in the interest rate. Maybe the result will only support the Fisher effect if the nominal interest rate increases at the same rate as the rate of inflation.

QUESTION FIVE

Soluton (A) Call option price						
Stock price now (p)			$60.00			
Exercise price of option (EX)			$60.00			
Number of periods to exercise in years (t)			0.5			
Compounded Risk-Free Interest Rate (rf)			2.45%			
Standard deviation (annualized) (s)			26%			
Call premium			$1.90			
d1			0.44846			
Delta N(d1) Normal Cumulative Density Function			0.67309			
d2			0.38346			
Delta N(d1) Normal Cumulative Density Function			0.64931			
SOLUTION B						
C=P+S0-X/(1+Rf)^T						
P= 4.74 -60+ (60/(1+2.45%)^0.5						
P=$4.02						
Therefore, put premium is			$4.02			

SOLUTION C: Long straddle and short straddle						
Cost for setting a long straddle						
Buy a call at strike price $60			-$1.90			
Buy a put at strike price $60			-$4.02			
Cost for setting a short straddle						
Sell a call at strike price $60			$1.90			
Sell a put at strike price $60			$4.02			
Net cost			$0.00			
Solution D net pay off when stock is at $50						
Strategy	stock price	call premium	put premium	stock payoff	Net payoff	
Long straddle	$50.00	-$1.90	-$4.02	$10.00	$4.08	
Short straddle	$50.00	$26.00	$4.02	-$10.00	$20.02	
Net profit /(loss)					$24.10	
Solution E Net payoff when stock is at $70						
Strategy	stock price	call premium	put premium	stock payoff	Net payoff	
Long straddle	$70.00	-$1.90	-$4.02	$10.00	$4.08	
Short straddle	$70.00	$1.90	$4.02	-$10.00	-$4.08	
Net profit /(loss)					$0.00	

QUESTION SIX

The term structure of interest is the variation in bond yields having similar profiles of risk for different bond terms. The relationship between the yield and maturity is the yield curve or sometimes known as effective maturity. The yield curve of a bond changes due to changes in demand and supply for medium-term, short term and long term bonds. An increase in interest rate will increase the demand for short term bonds faster than the demand for long term bonds

and hence the yield curve flattens. The characteristics of term structure of interest rates include volatility of short term bonds is higher than long term bonds, the long term bonds have higher yields than short term bonds and the changes in term bonds yields always move in the same direction.

The market segmentation theory states that the yield curve is determined by demand and supply of debt instruments that have different perception on maturities such as intermediate term, short-term, and long term. The difference in demand and supply in each market segment mentioned causes variation in bond prices and hence variation in yields. The difference in demand and supply of the bonds are caused by expected future interest rates and current interests rates. An increase in current rates will cause the future rates to decline causing increase in demand for long term bonds that lock high rates in the decreasing supply. Therefore, short term interest rate will be higher than short term rates. During decrease in current interest rates, the long term bonds will not be purchased by the bond buyers since bond prices will decline in future when interest rates increases. The supply for the bonds will increase because borrowers will be willing to lock in low rates. A higher supply and lower demand will cause a fall in the price of long term bond thus increasing their yield.

Expectation Hypothesis Theory posits that if future interest rates is expected to increase, there will be an upward sloping yield curve where higher yields will be realized in long term bonds. If there is an expectation for the future interest rates to decline, a lower yields will be realized on the long term bonds than short term bonds. The issuers of the long term bonds will not issue their bonds at higher rates if they also expect that in the near future, a lower interest rates will prevail. However, the expectation hypothesis has failed to explain why the yield of long term bonds are higher than short term bonds.

Liquidity premium theory explains the third characteristics of the term structure interest rates that states that high yielding bonds tend to have longer maturities. The increased risk as result of inflation risk and interest rate risk will lower the demand of the bonds and hence their yields will increase. The increase in yield due to increase in risk premium is what will compensate buyers for their increases risk. Liquidity Premium theory states that a bond that can easily be sold is more liquid thus reducing liquidity risk and that is why treasury bonds have low yields since they can be sold easily. Assets may be illiquid because supply exceeds demand or are riskier or it's more difficult to price or they have a stale previous sales or non-existent altogether. The liquidity premium is the risk premium that increases with the term of the bond. Therefore, we shall have upward sloping yield curve, even if we are expecting the future interest rate to decline by small margin or remain flat. Therefore, the upward sloping yield curve are well explained by liquidity premium theory of the term structure of interest rate in explaining bonds with different maturities.

QUESTION SEVEN

The unbiased expectation theory states that current long term interest rates have future short term interest rates that are implicit. It states that an investor should earn the same amount of interest rates from an investment in a two consecutive one year investment bonds as that person in a single two year bond today. The two year bond will have a higher interest rate compared to the two one year bonds. However, the Unbiased Expectation Theory states that by compounding the interest, the net income will be same.

5 year security = 2.65%

5 year one year security = 1.3%, 1.75%, 2.45%, 3.26% and 3.46%

BONDS AND RETURNS

The five year annual rate as follows:

RN= [(1+0.013) X (1+0.0175) X [(1+0.0245) X [(1+0.0326) X [(1+0.0346)] $^{1/5}$ -1

RN = [1.128133]$^{1/5}$ -1

RN = [1.0244]-1

RN= 0.0244 or 2.44%

The annual five year rate is 2.44%

The yield on five year yield is 2.65%

The short term yield will be lower compared to the long term yield. This means that for an investor to earn an equivalent return to todays 5 year bond, he will need to have invested in a one year bond today at 1.37% hoping that next year, the one year bond will increase to 2.44%. The long term interest rate of 2.65% is higher than short term interest rate of 2.44% implying interest rate will not increase in future as expected. The short term bonds will increase by 2.44%.

QUESTION EIGHT

Yield on 10yrs = 3.27%

Yield on 9yrs = 3.20%

Expectation is 3.32%

Implied forward yield on year 9 = (1.0327)^{10} /(1.032)^{9} -1

Implied forward yield on year 9 = 1.039- 1 = 3.9%

Expected rate in year 9 = 3.32%

Since the implied yield is higher than expected rate, the investor should long the bond.

QUESTION NINE

=MMULT(TRANSPOSE(A2:J61-A63:J63),A2:J61-A63:J63/9)

=A2:J61-A63:J63

BONDS AND RETURNS

IBM	**CVX**	**MRK**	**NEE**	**HMC**	**MS**	**MLSS**	**CYH**	**SNY**	**GOOG**
CVX	0.02318	0.021903	0.082523	-0.19475	0.326291	0.021628	0.12229	-0.0137	-0.13962
MRK	0.022893	0.020695	-0.00937	-0.02082	-0.09381	-0.10653	-0.06623	0.036069	0.003267
NEE	0.022577	0.020499	-0.03677	-0.226804	-0.22656	0.021147	-0.06049	0.052105	0.026778
HMC	-0.09821	-0.08489	-0.03873	0.013889	-0.04546	-0.01896	-0.0179	0.001278	0.052959
MS	0.022356	0.020048	-0.02867	0.246575	0.082011	0.020622	0.079935	-0.09821	-0.08489
MLSS	-0.07567	-0.07567	-0.05465	-0.131868	-0.18826	0.045466	0.026536	0.018942	0.013464

BONDS AND RETURNS

CYH	-0.0784	-0.0784	0.072996	-0.049873	0.006024	0.083729	0.086814	0.097071	0.080778
SNY	0.047489	0.047489	-0.03422	-0.014122	0.161677	-0.01176	0.003732	0.002478	0.011211
GOOG	-0.03359	-0.03359	-0.04628	0	-0.10825	0.042474	-0.02007	0.017136	0.027717

TRANSPOSE